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FURTHER MEASUREMENTS OF THE EFFECT OF PRESSURE ON RESISTANCE

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In a previous paper[1] data for the effect of pressures up to 12000 kg/cm^2 on the resistance of 22 metals were given. It has now been possible to extend the results to 18 more elements, and 6 alloys. The extension has been made possible by two changes in the technique. In the first place, by a change in the method of leading electrical connections into the pressure chamber it has been possible to replace the previous method of measuring resistance with a Carey Foster bridge by a potentiometer method. This makes it possible to measure accurately specimens whose total resistance is very low, and removes the restriction that the specimen must be in the form of a fine wire. In the second place, by a modification in the design of the apparatus, it has been possible to considerably extend the temperature range. The results now cover a range from atmospheric to 12000 kg. pressure, and an extreme temperature range from 0° to 275° C.

In selecting the substances to be measured over this increased range I have paid particular attention to the matter of liquid metals. Previous to this, the effect of pressure on both the liquid and solid state was not known for a single metal. The resistance of metals in the liquid state would seem to be particularly worthy of study, because here the crystalline structure introduces no complications. Six elements have now been investigated in the liquid and the solid states. Furthermore, I have endeavored particularly to investigate some of the more unusual elements, in the expectation that elements from unusual parts of the periodic table might show new types of behavior. This attempt has been rewarded by the discovery of three more elements whose pressure coefficient of resistance is positive; bismuth and antimony were the only ones known previously.

A number of commercial alloys have been included as being of some interest because of their wide use for scientific purposes. The measure-

ments on them had been already made for another purpose. The composition of the alloys is as follows:

Hoskins Mfg. Co.	Chromel A	Ni 80%, Cr 20%.
	Chromel B	Ni 85%, Cr 15%.
	Chromel C	Ni 64%, Cr 11%, Fe 25%.
Electrical Alloy Co.	Comet	Ni x%, Cr y%, Fe z%.
Driver Harris Co.	Therlo	Al 2%, Mn 13%, Cu 85 (this is like Manganin).
	No. 192 Alloy	Ni 30%, Cr 2%, Fe 68%.

Only a very rough summary of the principle results of the measurements can be attempted here. The results will be published in full detail elsewhere. In the table are shown the average pressure coefficients to 12000 kg., and also the initial coefficients at atmospheric pressure. The numerical values of the table may be supplemented by the following remarks on various significant aspects of behavior.

TABLE

EFFECT OF PRESSURE ON RESISTANCE

Substance	Mean Pressure Coefficient, 0–12000 kg.	Instantaneous Pressure Coefficient at 0 kg.	Substance	Mean Pressure Coefficient, 0–12000 kg.	Instantaneous Pressure Coefficient at 0 kg.
Li, solid, 0°	+0.0772	+0.0768	Bi, liquid, 275°	−0.0101[c]	−0.0123
Li, liquid, 240°	+0.0493	+0.0493	As, 0°	−0.0033	.....
Na, solid, 0°	−0.0345	−0.0663	W, 0°	−0.0135	−0.0143
Na, liquid, 200°	−0.0436	−0.0922	La, 0°	−0.0331	−0.039
K, solid, 25°	−0.0604	−0.0186	Nd, 0°	−0.0213	−0.0238
K, liquid, 165°	−0.0809[a]	−0.0168	Carbon, amorphous, 0°	−0.0100	−0.0118
Mg, 0°	−0.0408	−0.0447	Carbon, graphite, 0°	+0.047	+0.077
Ca, 0°	+0.0106	+0.0129	Si, 0°	−0.0117	.....
Sr, 0°	+0.0680	+0.0502	Black phosphorus, 0°	−0.081	−0.0200
Hg, solid, 0°	−0.0236[b]	.....	Chromel A, 0°	−0.0134	.....
Hg, liquid, 25°	−0.0219	0.0334	Chromel B, 0°	−0.0169	.....
Ga, liquid, 30°	−0.0531	−0.0640	Chromel C, 0°	−0.0427	.....
Ga, solid, 0°	−0.0247	.....	Comet, 0°	−0.0241	−0.0263
Ti, 0°	±0.01??	.....	Therlo, 0°	−0.0228	−0.0236
Zr, 0°	−0.040	−0.040	No. 193 alloy, 94°	−0.0179	.....

[a] Average 0–9000 kg.
[b] Average 7640–12000 kg.
[c] Average 0–7000 kg.

With regard to purity, Ga was prepared under the direction of Professor T. W. Richards for atomic weight work, and had less than 0.01% impurity. The Na, K, Mg, Hg, Bi, W, and black phosphorus were also of high purity. The Ca contained about 0.1% impurity, and the impurity in the Li and Sr was of the order of 1%. The Ti and Zr were known to be impure with 1.8% and 0.6% of W respectively; the impurity may have been higher. Judging by the temperature coefficients of resistance, As,

La, and Nd were considerably impure. The graphite was the best Acheson graphite, but the results were not reproducible. Similarly results with Si were not reproducible. All that can be expected of C and Si is the order of magnitude of the effect.

Normal Solids.—The substances with negative pressure coefficients of resistance are Na, K, Mg, Hg, Ga, Ti, Zr, As, W, La, Nd, Si, and black phosphorus. Mg and W were previously measured. It is now possible to give better values because of increased purity of the specimens available. La and Nd are the first metals in the rare earth group whose pressure coefficients have been measured; they show no novel features. Ti and Zr are also from a new region of the periodic table. The interesting feature of their behavior is the extreme smallness of the coefficient. Hg has not been previously measured in the solid state; its coefficient is somewhat greater than that of the liquid. Abnormal results were expected for gallium, because it expands when it freezes, but it was found instead to be quite normal. Arsenic might be anticipated to be abnormal because of its position in the periodic table relative to bismuth and antimony, but it turns out to be normal. Silicon, a non-metallic element, decreases in resistance, as is normal for metals, but the pressure coefficient becomes larger with increasing pressure and the temperature coefficient may reverse in sign at high pressures, both of which are abnormal features. Black phosphorus, also non-metallic, is remarkable for the very large size of the effect, the resistance decreasing under 12000 kg. to about 3% of its initial value. The relative coefficient, however, does not change so much as it does for some metals. Na and K are the first alkali metals whose pressure coefficients have been measured. They are remarkable for the largeness of the effect, which is larger than for any other metals as yet measured. Na decreases 40% and K 70% in resistance under 12000 kg. The pressure coefficient of these metals decreases greatly with increasing pressure and increases with increasing temperature, and the temperature coefficient decreases with increasing pressure. The metals previously measured have shown relatively little change in these coefficients.

Abnormal Solids.—Three new elements have been found whose resistance increases under pressure; these are Li, Ca, and Sr. This was a great surprise because all of these metals are highly compressible. With the single exception of the variation with pressure of the pressure coefficient of strontium, the behavior of these three metals is like that of Bi and Sb in that the instantaneous pressure coefficient increases with increasing pressure and decreases with increasing temperature, and the temperature coefficient falls with rising pressure.

Relative behavior of Resistance of Solid and Liquid.—The pressure coefficient of liquid bismuth is found to be negative and normal, although that of the solid is positive and abnormal. This points to the importance

of crystalline structure in determining the variations of resistance. On the other hand the coefficient of lithium is abnormal in being positive in both the liquid and solid states. (The melting curve of Li was measured under pressure and found to be normal in that the solid expands on melting.) The magnitude of the coefficient of the solid is less than that of the liquid. Gallium is normal in the solid as well as in the liquid, and the coefficient of the liquid is much larger than that of the solid. The coefficient of liquid mercury is slightly less than that of solid mercury. The coefficients of liquid sodium and potassium are less than those of the solids, but by only small amounts. The relative decrease in the coefficients of these solids with pressure may be greater than the decrease of the liquid. Liquid potassium has an abnormal change in the sign of the variation of temperature coefficient with pressure and the pressure coefficient with temperature.

All of the six metals measured in the solid and liquid state agreed in showing a relatively small change in the ratio of the resistance of liquid to solid along the melting curve from low to high pressure. Thus the change in the ratio of the resistance of liquid to solid potassium was from 1.56 to 1.55 under a pressure increase of 9700 kg. This pressure is sufficient to decrease the difference of volume between liquid and solid to 0.31 of its initial value. It seems to be universally true that the temperature coefficient of the liquid is less than that of the solid, and the change of resistance on melting is in the direction of the change of volume.

Theoretical Implications.—The following remarks are only two of many that might be suggested by the data.

It is probable, because of the peculiar nature of the atomic structure of lithium and the fact that its pressure coefficient of resistance is positive, that the picture of the mechanism of electrical conduction given by Wien and Lindemann as a space lattice of electrons sliding in the channels of a space lattice of atoms may have much of truth for this particular element.

The new evidence now at hand does not make it necessary to modify the former point of view[2] that for most elements the most important single factor in determining the variations of electrical resistance is the amplitude of atomic vibration.

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[1] Bridgman, P. W., *Proc. Amer. Acad., Boston*, **52**, 1917, (573–646).

[2] Bridgman, P. W., *Physic. Rev., Ithaca, N. Y.*, **9**, 1917 (269–289).

CHARACTERISTIC ABSORPTION OF X-RAYS: L SERIES

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Introduction.—Critical absorption wave-lengths associated with the L series of X-rays have been measured by E. Wagner[1] and M. de Broglie.[2] These scientists have investigated the spectra of ten of the chemical elements from tungsten to uranium, both inclusive. Each employed a photographic X-ray spectrometer with a rocksalt crystal. M. de Broglie found two critical absorption wave-lengths characteristic of each chemical element except, in the case of mercury, for which he gives only one. For bismuth, thorium and uranium he found a third, weak band.

In the investigation reported in this note we employed an ionization spectrometer with a calcite crystal. We found three critical absorption wave-lengths characteristic of each of the nine elements examined.

A brief discussion of the bearing of this new data on certain empirical laws and on recent theories of the mechanism of X-radiation may be found in the concluding paragraphs.

Apparatus and Method of Measurement.—A detailed description of the ionization spectrometer and the method of using it so as to obviate certain errors of measurement has been given in previous articles.[3] These articles also describe the plant for generating and controlling the X-radiation. Two X-ray tubes have been employed, each equipped with a Coolidge cathode and with a thin glass window blown in a side arm. One of these tubes contained a tungsten and the other a molybdenum target. The current exciting the tubes came from a high tension storage battery. The ionization chamber contained ethyl-bromide or methyl-iodide, and sometimes a mixture of both.

In measuring a critical absorption wave-length we place a thin sheet of the chemical element to be investigated (or of a salt containing it) in the path of the beam of rays between the X-ray tube and the first slit of the spectrometer. We then measure the ionization currents for a series of positions of the crystal corresponding to wave-lengths in the neighborhood of the critical values. From these measurements we platt curves representing the ionization current as a function of the crystal table angle (see figs. 1 and 2). At a critical absorption point a sharp drop appears on the curve, indicating that X-rays of shorter wave-length than the critical value are absorbed by the chemical element to a greater extent than X-rays of longer wave-length. For the two strongest absorption bands characteristic of each element we have obtained the curves on both sides of the crystal table zero. The angular distance between the mid-points on two corresponding drops gives us twice the glancing angle, θ,

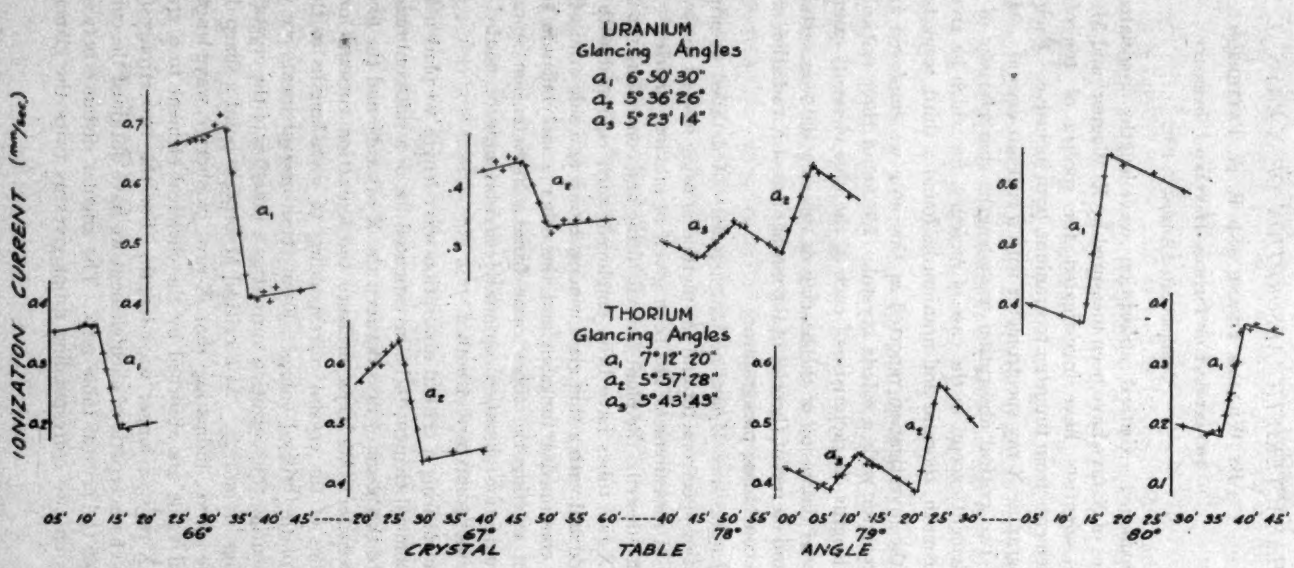
X-RAY ABSORPTION SPECTRA
L SERIES
URANIUM
Glancing Angles
a_1 6°50'30"
a_2 5°36'26"
a_3 5°23'14"
THORIUM
Glancing Angles
a_1 7°12'20"
a_2 5°57'28"
a_3 5°43'49"
IONIZATION CURRENT (mm/sec.)
CRYSTAL ANGLE
TABLE

FIG. 1

from which we compute the wave-length, λ, by means of the usual formula

$$\lambda = 2d \times \sin\theta,$$

where d is the distance between the reflecting planes of the crystal. The glancing angle for the third and weakest absorption band has been estimated by careful comparison with the neighboring stronger absorption drop obtained in the same series of readings.

Data Obtained.—Table 1 contains the results of our measurements. For purposes of comparison we include also the wave-lengths given by Wagner[1] and de Broglie.[2] The angular breadths of the drops vary from

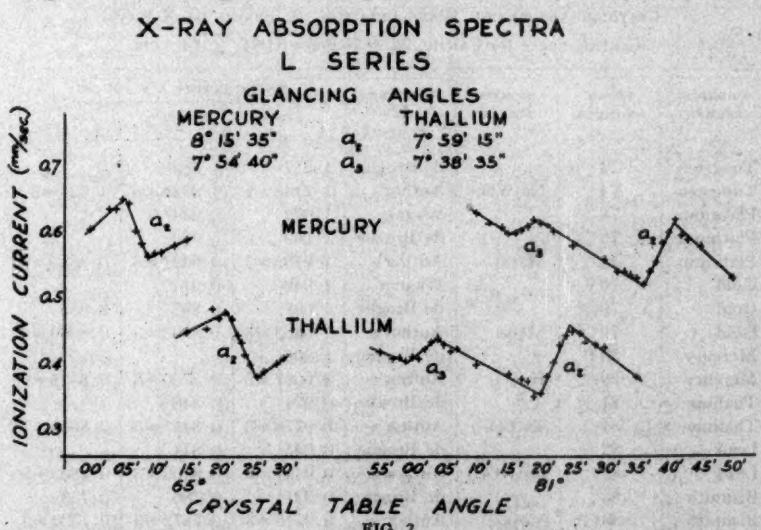

FIG. 2

three to ten minutes of arc according to circumstances, and we believe that we can estimate the positions of their centres to within ten to thirty seconds, depending upon the magnitudes of the drops and the regularity of the curves. Considerations of this kind have given us the estimates of the accuracy of each measurement recorded in the table.

Relative Positions of Emission and Absorption Wave-lengths.—In the article referred to above[3] the authors have called attention to the fact that in the case of tungsten: (*a*) the absorption wave-length, a_3, is slightly shorter than that of the highest frequency emission line of the L series, namely, γ_4; (*b*) the absorption wave-length, a_2, is a large fraction of one per cent longer than that of the emission line, γ_2; and (*c*) the absorption wave-length, a_1, is a large fraction of one per cent longer than the emission line, β_5. The measurements on which these conclusions were based were made under the same experimental conditions for both the emission and the absorption lines. We have not measured the L series of emission

lines for chemical elements of higher atomic numbers than that of tungsten with our spectrometer. Hence we cannot make a direct comparison between emission and absorption wave-lengths obtained under the same experimental conditions for these elements. If we take the wave-lenghts recorded by Siegbahn and Friman,[4] we find that (*a*) our value for the absorption wave-length, La_3, is shorter than that of the emission line, γ_4, for each element and that the difference between the two increases slightly with the atomic number; (*b*) our value for the absorption wave-length,

TABLE 1

CRITICAL ABSORPTION WAVE-LENGTHS. L SERIES OF X-RAYS

Grating Space for Calcite, $2d = (6.056 \pm 0.004) \times 10^{-8}$ cm.

CHEMICAL ELEMENT	ATOMIC NUMBER	ABSORBING SCREEN	MEASURED BY	WAVE-LENGTHS, $\lambda \times 10^8$ CM.		
				La_1	La_2	La_3
Tungsten	74		de Broglie	1.217	1.083	
Tungsten	74	Na_2WO_4	Author's	1.2136±1	1.0726±5	1.024±3
Platinum	78		Wagner	1.072	0.934	
Platinum	78		de Broglie	1.069	0.930	
Platinum	78	Metal	Author's	1.0705±3	0.9321±3	0.8885±9
Gold	79		Wagner	1.042	0.914	
Gold	79		de Broglie	1.038	0.898	0.858
Gold	79	Metal	Author's	1.0383±3	0.8993±3	0.8606±8
Mercury	80		de Broglie	1.006		
Mercury	80	$HgSO_4$	Author's	1.0067±5	0.8700±3	0.8335±9
Thalium	81		de Broglie	0.974	0.840	
Thalium	81	$Tl_2(SO_4)_3$	Author's	0.9776±3	0.8415±3	0.8055±14
Lead	82		de Broglie	0.945	0.811	
Lead	82	$Pb(NO_3)_2$	Author's	0.9497±3	0.8133±3	0.7803±9
Bismuth	83		de Broglie	0.921	0.786	0.753
Bismuth	83	Metal $Bi(NO_3)_3$	Author's	0.9216±3	0.7872±3	0.7532±9
Radium	88		de Broglie	0.802	0.670	
Thorium	90		de Broglie	0.757	0.624	0.604
Thorium	90	$Th(SO_4)_2$	Author's	0.7596±3	0.6286±3	0.6044±7
Uranium	92		de Broglie	0.718	0.588	0.564
Uranium	92	Acetate	Author's	0.7214±3	0.5918±3	0.5685±7

La_2, is longer than that of the emission line, γ_2, for tungsten and gold, but shorter for the other elements; (*c*) our value for the absorption wave-length, La_1, is longer than that of the emission line, β_5, for tungsten, gold, and thallium, but shorter for platinum and bismuth. The relative positions of these emission and absorption lines appear to be matters of considerable theoretical importance,[5] and this subject will be discussed at greater length in a future note.

The Wave-lengths as Functions of the Atomic Numbers.—Many of the laws connecting X-ray wave-lenghts with each other, or with the atomic numbers of the chemical elements may be expressed most simply in terms

of the wave-numbers, $1/\lambda$, or of the ratios of the frequencies of vibration to the Rydberg fundamental frequency for heavy atoms. In his classical memoirs on the wave-lengths of characteristic X-rays Moseley called attention to the fact that the square root of the wave-number (or frequency) of a line in the X-ray spectrum of an element is very nearly a linear function of its atomic number. To test this relation in the case of critical

TABLE 2

CRITICAL ABSORPTION WAVE-NUMBERS. L SERIES OF X-RAYS

Values of $1/\lambda$ and of $\sqrt{1/\lambda}$

CHEMICAL ELEMENT	ATOMIC NUMBER	$1/\lambda$	$\sqrt{1/\lambda}$	DIFFERENCES
			L_{α_1}	
Tungsten	74	0.8240±1	0.9077	
Platinum	78	0.9341±3	0.9665	0.0147±1
Gold	79	0.9631±3	0.9814	0.0149±3
Mercury	80	0.9933±4	0.9966	0.0152±4
Thalium	81	1.0229±3	1.0114	0.0148±4
Lead	82	1.0530±3	1.0262	0.0148±3
Bismuth	83	1.0851±3	1.0417	0.0155±3
Thorium	90	1.3164±5	1.1473	0.0151±1
Uranium	92	1.3862±5	1.1774	0.0151±2
			L_{α_2}	
Tungsten	74	0.9323±5	0.9656	
Platinum	78	1.0728±4	1.0358	0.0176±1
Gold	79	1.1120±4	1.0545	0.0187±4
Mercury	90	1.1494±4	1.0721	0.0176±4
Thalium	81	1.1883±5	1.0901	0.0180±5
Lead	82	1.2295±5	1.1088	0.0187±5
Bismuth	83	1.2703±4	1.1271	0.0182±5
Thorium	90	1.5908±8	1.2613	0.0192±1
Uranium	92	1.6898±8	1.2999	0.0193±3
			L_{β_2}	
Tungsten	74	0.977±3	0.988	
Platinum	78	1.1255±12	1.0609	0.0182±5
Gold	79	1.162±1	1.0780	0.0171±11
Mercury	80	1.200±1	1.0953	0.0174±11
Thalium	81	1.242±2	1.1142	0.0189±15
Lead	82	1.282±2	1.1320	0.0178±16
Bismuth	83	1.328±2	1.1523	0.0203±13
Thorium	90	1.655±2	1.2863	0.0192±2
Uranium	92	1.759±2	1.3263	0.0200±7

absorption data we have calculated the wave-numbers and the square roots of the wave-numbers for all of our values. Column 3, in Table 2, contains the wave-numbers and Column 4 their square roots. In Column 5 we have tabulated the increase in the value of $\sqrt{1/\lambda}$ per unit increase in the atomic number. It appears that these differences do not vary from

the average values of the differences in their neighborhoods by more than the estimated errors, except in one or two cases. In other words, the $\sqrt{1/\lambda}$ may be represented as a function of the atomic number by a smooth curve, from which the individual values do not differ by more than the experimental errors. In the case of the absorption band, La_1, this curve differs a small amount from a straight line. In the cases of the bands La_2 and La_3, however, the departures from the straight line law appear to be much greater. A very much more pronounced variation from the linear law has been noted in the case of the critical absorption frequencies associated with the K series of X-rays.[6] These departures from the straight line law have been ascribed to the relativity change of mass with velocity of the electrons connected with the production of the X-rays.

The fact that the values of $\sqrt{1/\lambda}$ do not differ from a smooth curve law by more than the indicated errors of measurement shows that our estimates of accuracy are substantially correct.

The Relation between the Emission and Absorption Frequencies.—It has been found by experiment that the difference between the critical absorption wave-number of the K series and one of the critical absorption wave-numbers of the L series equals the wave-number of one of the α lines in the K series to within the limits of error.[7] The experiments were performed with the X-ray spectrum of tungsten. As the emission wave-lengths in the K series have not been measured for chemical elements of higher atomic numbers than that of tungsten, we cannot test this law between the K and L series for the heavier elements. We can test the law, however, in the case of the L *and* M series of thorium and uranium, for here all the ncessary data is available. The tables of Siegbahn and Friman[4] contain the emission wave-lengths in the L series, and Stenström[8] has measured three absorption wave-lengths in the M series for each of these two elements. Using our values of the L absorption wave-lengths we have compiled the following table of wave-numbers (Table 3):

TABLE 3
RELATION BETWEEN ABSORPTION AND EMISSION WAVE-NUMBERS
L and M Series

		U	Th		U	Th		U	Th
Absorption	La_1–Ma_1	1.099	1.047	La_1–Ma_2	1.085	1.034	La_1–Ma_3	1.038	0.988
Emission	$L\alpha_1$	1.098	1.045	$L\alpha_2$	1.085	1.032			
Absorption	La_2–Ma_1	1.403	1.322	La_2–Ma_2	1.389	1.309	La_2–Ma_3	1.342	1.263
Emission	$L\beta_3$	1.408	1.319	$L\beta_1$	1.389	1.305			
Absorption	La_3–Ma_1	1.472	1.385	La_3–Ma_2	1.458	1.372	La_3–Ma_3	1.411	1.326
Emission							$L\beta_2$	1.408	1.319

It appears from this table that the wave-numbers of the emission lines $L\alpha_1$ $L\alpha_2$ and $L\beta_1$ agree very well with the differences between the absorption wave-numbers La_1–Ma_1, La_1–Ma_2 and La_2–Ma_2, respectively.

Judging from this data alone, there is some question as to whether the emission line $L\beta_3$ belongs to the absorption difference La_2–Ma_1 or to La_3–Ma_3.

No emission lines have been observed in the spectrum of either thorium or uranium that correspond to the other absorption differences contained in the table.

Four emission lines have been found by Siegbahn and Friman in the spectra of both thorium and uranium in addition to those contained in our Table 3, namely, $1, \beta_2, \gamma_1$, and $\gamma_{2,3}$. The way in which the frequencies of the lines for different chemical elements vary with their atomic numbers[4] and the researches of Webster and Clark[9,5] on the voltages required to produce the lines in the spectrum of platinum indicate that 1 and β_2 belong to the same X-ray group as α_1 and α_2, and therefore also to the absorption La_1. Similarly, γ_1 belongs to the absorption La_2. If these lines obey the frequency difference law, there must be critical absorption wave-lengths at $\lambda = 2.37 \times 10^{-8}$ cm. and at $\lambda = 2.23 \times 10^{-8}$ cm. for thorium and uranium, respectively, to correspond with the emission line 1. These wave-lengths lie within the range of possible measurements, but the spectrum in their neighborhood has not been examined for critical absorption. Similarly, there should be critical absorption wave-lengths at $\lambda = 16.2 \times 10^{-8}$ cm. for thorium and at $\lambda = 15.8 \times 10^{-8}$ cm. for uranium to correspond with the emission lines β_2 and γ_1. These wave-lengths, however, lie beyond the range of wave-lengths that can be measured at present by means of crystal spectrometers.

It is well known that Bohr's theory of radiation explains very simply the above relation between emission and absorption wave-numbers. The relation, however, should not be absolutely exact, if, as Sommerfeld supposes, some of the atoms contain M orbits that are elliptic, and other atoms of the same chemical element contain M orbits that are circular. In this case the L absorption drops should have a complex structure. We have not been able to observe such a structure in the drops on our curves, and the effect must be exceedingly small, if it exits at all.

Theoreies have been developed by Rubinowicz[10] and Bohr,[11] according to which transfers of electrons between certain pairs of orbits cannot take place.[12] If we apply these theories to the data contained in Table 3, we find that according to both theories the transfers represented by Ma_1–La_1 and Ma_2–La_2 should occur. As a matter of fact, the corresponding lines, α_1 and β_1, are the strongest lines in the L series. According to Rubinowicz's theory the transfer Ma_2–La_1 should occur, but according to Bohr's theory it should not, unless the atom is in a field of force. The corresponding line, α_2, has been observed, but is relatively very faint. According to both theories the transfer Ma_1–La_1 should not occur, and, as noted above, the experiments do not indicate conclusively whether the line β_3 belongs to this combination or not. Both theories allow the transfer

Ma_s–La_1, but no line corresponding to it has been observed. The transfer Ma_s–La_2 can occur according to Rubinowicz's theory, but not according to Bohr's, and there is no line to correspond with it. The other transfers, involving La_2, are not covered by the theory, for it does not in its present form include a third L orbit.

Sommerfeld's L-Doublet Theory.—Sommerfeld has worked out an interesting theory of the splitting up of the L series into two distinct parts—the L series doublet. An account of his reasoning may be found in his book, Atombau und Spektrallinien, Chapter 5. According to this theory the difference in frequency $\Delta\nu$ between the two parts of the L series is due to the difference between the energies of electrons travelling in circular orbits and of electrons travelling in elliptic orbits. Some atoms are supposed to have circular L orbits, and other atoms of the same chemical element, elliptic L orbits. The general equation (28), in the chapter referred to, for the difference, $\Delta\nu$, in wave-number, reduces to

$$\frac{\Delta\nu}{\nu_\infty} = \frac{1}{\alpha^2}(\sqrt{2}\sqrt{1 + \sqrt{1 - b^2}} - \sqrt{4 - b^2}), \qquad (1)$$

for the L doublet, where $\nu_\infty = 109737$ is Rydberg's fundamental wave-number, $\alpha = \frac{2\pi e^2}{hc} = .007286$, $b = \alpha(N - n)$, N is the atomic number, and n, a correction term due to the influence of neighboring electrons in the atom. Sommerfeld has tested his theory by means of the wave-number differences between certain emission lines in the spectra of a great many chemical elements, and has deduced from this data the value $n = 3.63$. Although there is some difficulty in interpreting this value of the constant, n, we have used it to test the formula by means of our values of the critical absorption wave-numbers.[13] According to the theory the wave-number difference in the doublet should equal the difference between the wave-numbers of the absorption bands La_1 and La_2.

TABLE 4

THE L-DOUBLET WAVE-NUMBER DIFFERENCES, $\Delta\nu \times 10^{-3}$

CHEMICAL ELEMENT	ATOMIC NUMBER	OBSERVED	CALCULATED		
			$n = 3.63$	$n = 3.45$	MODIFIED
Tungsten	74	0.1083±6	0.1071	0.1087	0.1037
Platinum	78	0.1387±7	0.1369	0.1384	0.1327
Gold	79	0.1489±7	0.1453	0.1468	0.1411
Mercury	80	0.1561±8	0.1540	0.1558	0.1496
Thalium	81	0.1654±8	0.1635	0.1652	0.1587
Lead	80	0.1765±8	0.1733	0.1751	0.1682
Bismuth	83	0.1852±7	0.1836	0.1856	0.1782
Thorium	90	0.2744±13	0.2713	0.2739	0.2639
Uranium	92	0.3036±13	0.3023	0.3044	0.2941

Table 4 contains the observed wave-number differences of the L doublet, with estimated errors of experiment, and also three sets of computed differences. The values given in Columns 4 and 5 have been calculated directly by the above formula (1) without expanding it into a series of ascending powers of b. It would be necessary to use six or more terms of such a series in order to attain the indicated accuracy.

The differences between the observed values and those computed, using Sommerfeld's value of $n = 3.63$, all have the same sign, and are distinctly greater than the errors of measurement. The values obtained with $n = 3.45$ compare more favorably with the experimental results, and indicate that with $n = 3.43$ the formula would give the observed differences, $\Delta\nu$, within the limits of error for nearly all the elements contained in the table. Even in this case there appears to be a small systematic variation between the theoretical and experimental frequency differences, as though the formula were nearly but not quite correct.

The constant n represents the repulsive force on an electron in the L orbit due to the electrons in the K orbit plus the force due to the other electrons in the L orbit. If n_1 and n_2 are the numbers of electrons in the K and the L orbits, respectively, $n = n_1 + s_{n_2}$ for circular orbits, where

$$s_n = \frac{1}{4} \sum_{s=1}^{s=n-1} \operatorname{cosec} \frac{s\pi}{n}.$$

The values of n for elliptic orbits is assumed equal to that for circular orbits. There are, however, no values of n_1 and n_2 that agree exactly with the empirical values $n = 3.63, 3.45$ or 3.43. For $n_1 = 2$ and $n_2 = 5$ we have $n = 3.38$. Other evidence of the distribution of electrons in atoms indicates that this arrangement is highly improbable.

As a matter of fact, the theory in its present form does not take account entirely of the influence of all the electrons in the L orbit itself. When one electron is removed from this orbit, the other electrons change their positions relative to each other and to the nucleus. In calculating the change in the atom's energy envolved we must take account of the change in their energy also. A modification of Sommerfeld's formula which includes these energy changes may be obtained by taking the product of n_2 into the value of the right-hand member of equation (1), with $n = n_1 + s_{n_2}$, and subtracting from it the product of $n_2 - 1$ into the value of the same expression, with $n = n_1 + s_{n_2-1}$. Before computing wave-number differences from this modified formula we must assign definite values to n_1 and n_2. This gives a sort of theoretical value for n.

Column 6, in Table 4, contains wave-number differences calculated in this way with $n_1 = 2$ and $n_2 = 4$. They differ uniformly from the experimental results by about 3%.

The above suggested alterations in the theory, however, do not obviate

certain difficulties (discussed by Sommerfeld in his book) that we encounter, if we suppose that the orbits all lie in the same plane. Possibly these difficulties will disappear, when we study more thoroughly atomic models in which the orbits do not lie in the same plane.

[1] E. Wagner, *Ann. Physik, Leipzig*, March, 1915 (868).

[2] M. de Broglie, *J. Physique, Paris*, May-June, 1916 (161); and *Paris C. R. Acad. Sci.*, Nov. 24, 1919 (962).

[3] Blake and Duane, *Physic. Rev., Ithaca*, Dec., 1917 (624); and Duane and Patterson, *Ibid.*, 1920.

[4] Siegbahn and Friman, *Phil. Mag., London*, April, 1916 (423); and Nov., 1916 (497).

[5] D. L. Webster, *Proc. Nat. Acad. Sci., Washington, D. C.*, Jan., 1920 (26).

[6] Duane and Hu, *Physic. Rev.*, June, 1918 (488); and Dec., 1919 (516); and Duane and Shimizu, *Ibid.*, Feb., 1919 (159) and Dec., 1919 (522).

[7] Duane and Shimizu, *Ibid.*, July, 1919 (67); Duane and Patterson, *Ibid.*, 1920; Duane and Stenström, *Ibid.*, April, 1920 (302); and *Proc. Nat. Acad. Sci.*, Aug. 1902.

[8] Stenström, *Doctor's Dissertation*, Lund, 1919.

[9] Webster and Clark, *Proc. Nat. Acad.*, March, 1917 (181).

[10] Rubinowicz, *Physik. Zs., Leipzig*, **19**, 1918 (441–465).

[11] Bohr, *Copenhagen Academy*, 1918.

[12] Compare Sommerfeld, *Atombau und Spektrallinien*, Chapter 6.

[13] Compare de Broglie *Paris, C. R. Acad. Sci.*, Nov. 24, 1919 (962).

ON THE RELATIVE POSITIONS AND INTENSITIES OF LINES IN X-RAY SPECTRA

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Grouping of L Series Lines.—The wave-lengths of the lines in the *L* series of X-rays have been measured for a great many of the chemical elements by Siegbahn and Friman.[1] They have published graphs which represent the square roots of the frequencies of corresponding lines in the spectra of different elements as functions of the atomic number. The graphs for the lines 1, α_2, α_1, β_2 and β_5 are very nearly straight, indicating that the square roots of the corresponding frequencies are almost linear functions of the atomic numbers, whereas the graphs for the other lines are curved, indicating a marked departure from the linear law. This would seem to mean that we can divide the lines in the *L* series into at least two groups.

In his work on characteristic absorption de Broglie[2] found three critical absorption wave-lengths associated with the *L* series of gold, bismuth, thorium and uranium. The authors[3] have extended this, and have measured the third critical absorption wave-length as well as the other two for the elements they examined from tungsten to uranium both inclusive. The fact that three critical absorption wave-lengths appear in the *L* series of

each chemical element would seem to indicate the existence of at least three distinct groups of lines.

The researches of D. L. Webster and Harry Clark[4] on the voltages required to produce the various lines in the L series of platinum and on the way in which their intensities increase with increasing voltage confirm this, and prove further that the lines η, β_1, γ_1 and probably γ_2 belong to the second group.

The modern theory of line spectra, also, seems to indicate the splitting up of the L series into groups. The differences in frequency between certain lines in the first group and corresponding lines in the second group are very nearly equal to each other. These pairs of lines with nearly the same frequency interval are η–1, β_1–α_2, γ_1–β_2 and γ_2–β_5. Sommerfeld[5] explains this division of the L series into groups with a constant frequency difference on the assumption that one group of lines is due to electrons falling into an L orbit that is circular, and that the other group is produced by electrons falling into an L orbit that is elliptic. The equation he derives from his theory for the magnitude of the frequency difference fits the facts with considerable precision.

The following facts indicate that the critical absoprtion La_1 belongs to the first group of lines, and that the critical absorption La_2 belongs to the second group of lines. Firstly, the wave-length of the critical absorption La_1 lies not far from that of the line of shortest wave-length in the first group, and the wave-length of La_2 lies close to that of the line of shortest wave-length in the second group. Secondly, as the authors have shown,[3] the square root of the frequency of La_1 for the different elements is almost a linear function of the atomic number, while that of La_2 departs from the linear law in the same way as do those of the second group of lines in Siegbahn and Friman's graphs. Thirdly, the frequency difference between La_2 and La_1 equals to a high degree of precision that between the pairs of lines η–1, β_1–α_2 and γ_2–β_5 which were all measured by the authors[6] under the same experimental conditions for the tungsten spectrum. Fourthly, the frequency differences between La_2 and La_1 for the various chemical elements from tungsten to uranium[3] agree very well with Sommerfeld's formula. Fifthly, the values of the critical voltages of two of the groups of lines in the L series of platinum measured by Webster and Clark[4] (namely, 13.20 kilovolts and 11.45 kilovolts) agree very well with the values of the same quantities calculated by the quantum equation from our critical absorption wave-lengths (namely 13.26 kilovolts and 11.54 kilovolts).

Relative Position of Lines.—It has been shown by Duane and Hu[7] that the critical absorption wave-length associated with the K series of Rhodium is about one-third of a per cent shorter than that of the shortest line (the γ line) in that K series. Further, Duane and Stenström[8] have found that the difference between the wave-length of the critical absorption in

the K series of tungsten and that of its γ line amounts to about one-half of one per cent, the γ line having the longer wave-length. In the case of each element the wave-lengths were measured under exactly the same experimental conditions, so that there can be no doubt but that the critical absorption wave-length in the K series is shorter than the wave-length of the γ line by an amount that considerably exceeds the errors of experiment.

Turning to the L series we find that the third critical absorption wave-length, La_3, appears to be shorter than that of the line of shortest wave-length in the L series, namely, γ_4.[3] The authors[6] have found the following values for these wave-lengths, $La_3 = 1.024 \pm 3$ and $\gamma_4 = 1.0261 \pm 6$, in the spectrum of tungsten. In the other two groups, however, the critical absorption wave-lengths are longer than those of the shortest wave-length lines respectively. In the tungsten spectrum the authors[3,6] found for the wave-lengths in the first group $La_1 = 1.2136 \pm 1$ and $\beta_8 = 1.2040 \pm 7$, and in the second $La_2 = 1.0726 \pm 5$ and $\gamma_2 = 1.0655 \pm 4$. In each case the difference between the two wave-lengths considerably exceeds the estimated errors of experiment.

As the relative position of these emission and absorption lines appears to be a matter of considerable theoretical importance the authors have repeated their experiments in a slightly different form. To make doubly sure of the facts they have taken the readings so as to show the position of the critical absorption and those of the emission lines on each side of it on one and the same curve. The experimental conditions were exactly the same as those in the experiments described in the article referred to,[6] except that the X-ray tube was turned so that the rays left the target making a slightly smaller angle with its surface than in the earlier experiments. This increased the absorption of the rays by the target itself, and made the absorption drop in the curves more prominent.

The curve in figure 1 represents the intensity of the radiation (measured by the currents in the X-ray spectrometer's ionization chamber) as a function of the crystal table angles. The drop in the curve marked a_1 corresponds to the critical absorption belonging to the first group of L series emission lines. The absorption of its own rays by the tungsten target alone produced this drop, as no absorbing screen was introduced in the path of the rays in this experiment. The tall peak on the curve represents the relatively strong emission line β_2. Its separation from the absorption drop corresponds very well with the wave-length interval between the two, calculated from the data obtained in the earlier experiments, namely 0.0283 ± 2. The small hump in the curve on the other side of the absorption drop represents the emission line, β_8.

This experiment proves conclusively that the critical absorption lies between the emission lines β_2 and β_8, in other words it has a longer wave-length than that of β_8.

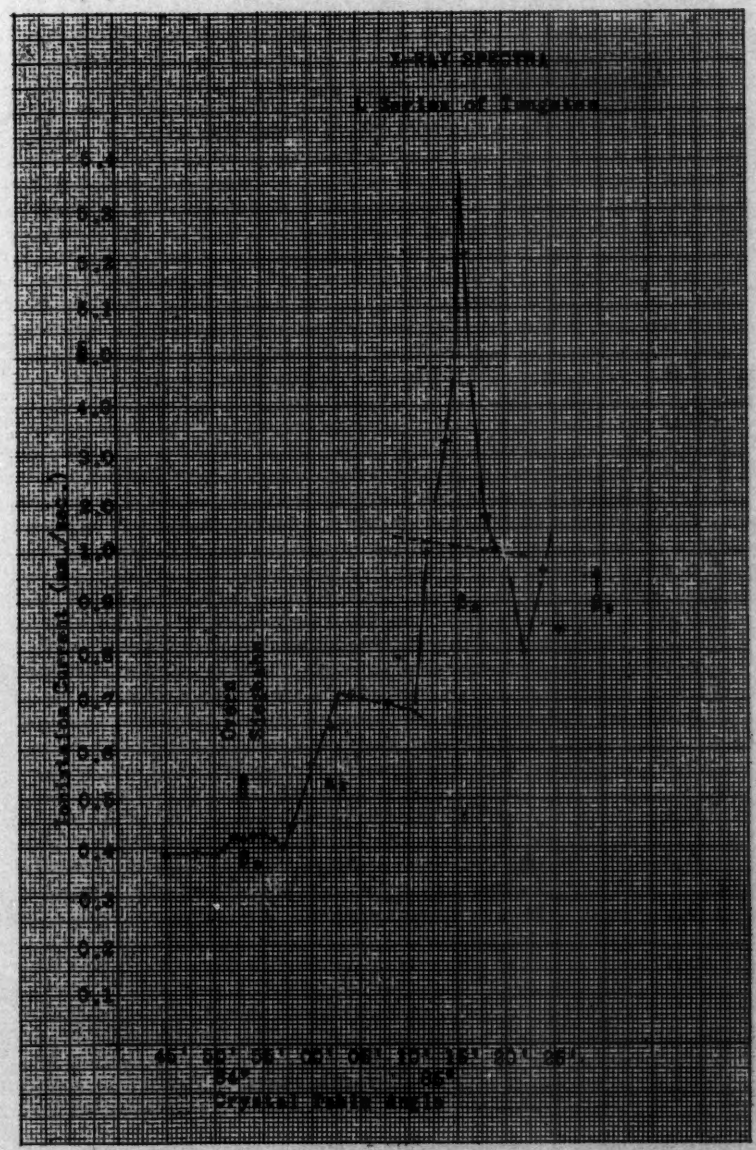

FIG. 1

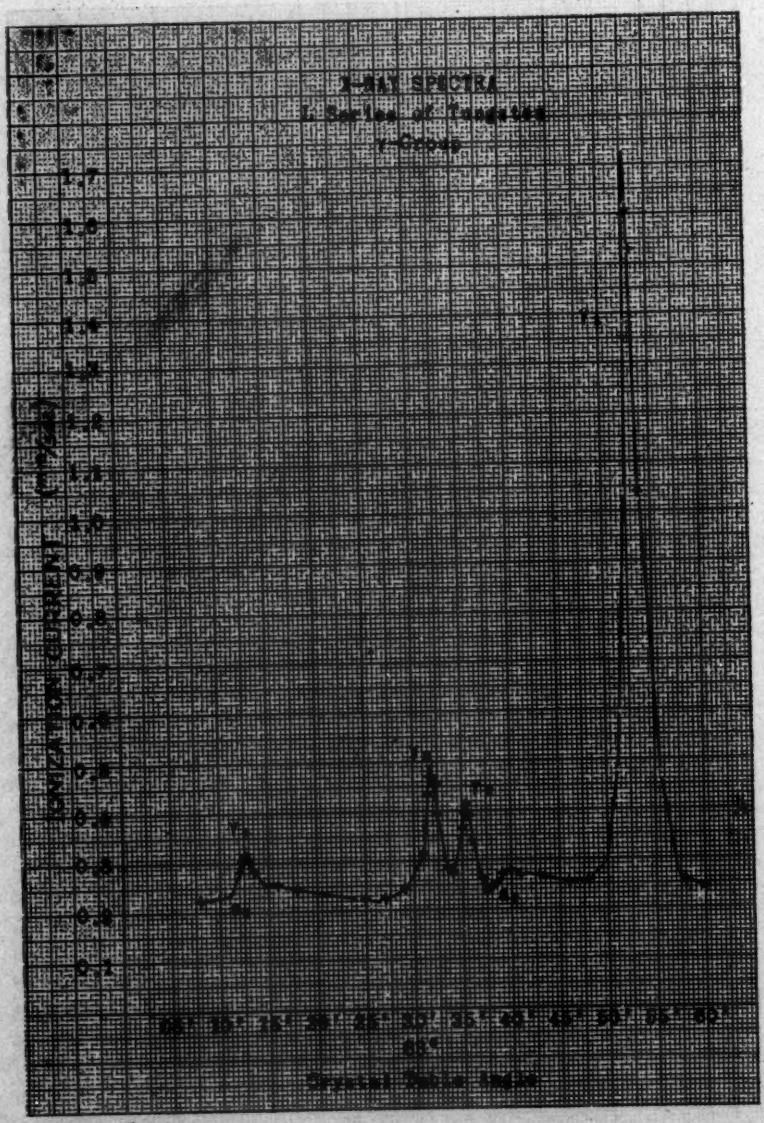

FIG. 2

The positions of β_5 as determined from the measurements of Overn[9] and Siegbahn[10] are marked above the curve in figure 1. According to their measurements this faint line has a wave-length a small fraction of one per cent shorter than our value, which would bring it still further from the critical absorption in the direction of shorter wave-lengths.

The curve in figure 2 represents the spectrum of tungsten in the neighborhood of the critical absorption La_2. The drop corresponding to the absorption and the peaks corresponding to the γ emission lines appear on the curve. The critical absorption lies between the emission lines γ_1 and γ_2, and therefore has a wave-length longer than that of γ_2. Overn's and Siegbahn's values for γ_2 are in exact agreement with our values.

According to the conception of radiation held by many scientists the critical absorption wave-length corresponds to the short wave-length limit of the group or series of emission lines with which it is associated. If this be true, or, to speak more accurately, if the term "group of lines" is defined in such a way that this is true, then β_5 cannot belong to the first group in the L series of X-rays, and γ_2 cannot belong to the second group.

The strongest evidence in favor of believing that β_5 belongs to the same *atomic mechanism* that produces the lines in the first group appears to be that the square root of its frequency increases from atom to atom nearly as a linear function of the atomic number, corresponding in this respect to all the lines in the first group but not to those in the second and third groups. In terms of the theory of atomic orbits this means that electrons falling into the L_1 orbit produce the line β_5, as they do all the lines in group 1. Webster and Clark[4] found that β_5 in the platinum spectrum appeared at a lower voltage than that required to produce the lines in the second group. This proves conclusively that β_5 cannot belong to the second group. The experiments on critical potentials, however, are not sufficiently accurate to decide whether β_5 appears at *exactly* the same voltage as the other lines in group 1, for β_5 is a weak line, and the difference between its wave-length and that of the critical absorption L_1 amounts to only 0.7% for tungsten. We are not, therefore, compelled to assume that β_5 can be produced by electrons in the X-ray tube having quantities of energy less than that given by the quantum equation, $Ve = h\nu$.

The argument in favor of supposing that the line γ_2 belongs to the mechanism that produces the lines in the second group rests largely upon the fact that for the various chemical elements the difference in frequency between γ_2 and β_5 equals the frequency interval between the lines in the other pairs belonging respectively to the two groups. It also equals the difference in frequency between the two critical absorptions La_2 and La_1, and is given quite accurately by Sommerfeld's L doublet formula. Hence, according to the theory of electron orbits, γ_2, in common with the

other lines belonging to the second group, is produced by electrons falling into the second, L_2, orbit.

On the experimental and theoretical evidence we have reached the conclusion that the emission line β_5 belongs to the atomic mechanism that produces the lines in the L_1 group, and that γ_2 belongs to that producing the lines in the L_2 group. On the other hand our experiments prove conclusively that the wave-length of β_5 is shorter than that of the La_1 critical absorption, and that the wave-length of γ_2 is shorter than that of La_2. A somewhat similar situation occurs in the spectra of ordinary light. Here we have resonance and ionization potentials. There is a difference, however, between the character of the absorption of X-rays and that of ordinary light.

The best way of explaining these phenomena may be somewhat as follows: Let us suppose that the critical absorption La_1 corresponds to the transfer of an electron from the L_1 orbit to the periphery of the atom—to an orbit there where it can find a place to stick. This does not mean a transfer to an indefinitely great distance from the centre of the atom. There may be many orbits outside that at the periphery, even in addition to those that belong to visible spectra. With the electron in the orbit at the periphery the atom is in a state such that by the transfer of an electron back to fill the vacancy in the L_1 orbit it can radiate any one of the emission lines 1, α_2, α_1, β_2, etc., but not β_5. If, now, the frequency of the incident X-ray beam is higher than that of the critical absorption, if the energy $h\nu$ is greater than that required to lift the electron to the periphery of the atom, it may carry the electron to an orbit further out or even outside of the atom altogether. In this case the atom would be in a state such that the return of an electron from one of these outside orbits would emit X-radiation of higher frequency and shorter wave-length than that of the critical absorption. This would correspond to the emission line β_5. A similar explanation applies also to the emission line γ_2, and the L_2 orbit.

From this point of view we would expect the critical potential for β_5 to be slightly higher than that for the other lines belonging to the L_1 orbit, and the critical potential for γ_2 to be slightly higher than that for the lines associated with the L_2 orbit.

Further, on this theory, the critical absorption does not correspond to the limit of the group, or series. It becomes incorrect to speak of the "absorption limit," as these words are usually employed.

An accurate, quantitative test for our point of view cannot be obtained without making special assumptions. According to our measurements, however, the difference in frequency between the emission line and the critical absorption for tungsten, each divided by the Rydberg constant, is 6.0 for β_5–La_1, and 5.6 for γ_2–La_2. These are of the order of magnitude of frequencies associated with the peripheries of atoms.

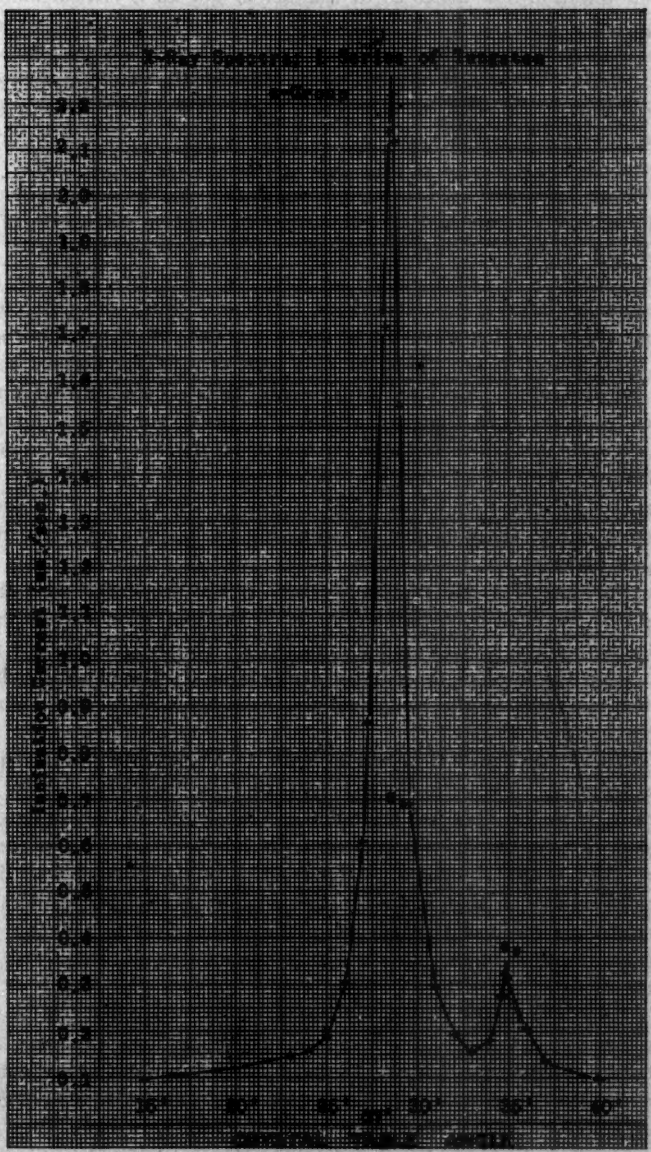

FIG. 3

The question now arises as to whether or not an L critical ionization frequency equals an L critical absorption frequency. A critical ionization frequency represents the transfer of an electron completely *outside* of an atom. In the case of the K series of iodine the two frequencies have been found to be equal to each other within the limits of error of the measurements.[7] Experiments are in progress to test this point for the L series.

Relative Intensities of Lines.—The ionization spectrometer provides us with an excellent method of estimating the relative intensities of lines in X-ray spectra. Accurate estimates of the relative intensities, however, cannot be obtained unless the lines lie fairly close together. If a considerable interval separates them, corrections must be applied for the changes with varying wave-length in the amounts of energy absorbed by the substances through which the rays pass, etc. These corrections cannot be accurately calculated at present. Special precautions must be taken to make sure that no critical absorption nor critical ionization wave-length lies between the wave-length of the lines to be compared with each other. These limits to the accuracy of relative intensity measurements present themselves in photographic spectrometry in addition to those due to the difficulty of estimating the blackening of the photographic plates.

In the case of a line that is not perceptibly broadened we take the height of the peak corresponding to it on the ionization current crystal table angle graph to represent its intensity. This height must be measured from the level of the curve corresponding to the general radiation on the two sides of the peak, and not from the axis of zero ionization current.

Two peaks appear on the curve in figure 1, corresponding to the spectral lines β_2 and β_6. The ratio of the heights of these two peaks is 116. This number, however, does not represent accurately the relative intensity of β_2 to β_6, for the critical absorption of La_1 lies between the two emission lines. The target absorbs a greater fraction of the X-ray energy in the neighborhood of β_6 than it does in the neighborhood of β_2. In this case, therefore, the ratio of the heights of the peaks gives us only an upper limit for the ratio of the intensities. β_2 cannot be more than 116 times as intense as β_6.

The curve in figure 2 contains four peaks corresponding to the emission lines γ_1, γ_2, γ_3 and γ_4. The relative heights of these peaks are represented by the numbers 100, 14, 18 and 6. These numbers have not been corrected for the errors due to absorption, etc., so that they give only an approximate estimate of the relative intensities of the emission lines. The lines γ_2 and γ_3, however, lie very close together, and the correction must be very small. Since the critical voltages for all the γ lines are not quite the same, their relative intensities depend somewhat on the voltage

applied to the X-ray tube. This was 22,750 volts. The applied voltage lies so far above the critical voltages, however, that the effect due to variations in the latter do not amount to as much as three per cent.

Figure 2 in the paper referred to above[6] contains curves representing the four strongest β lines. The heights of the peaks corresponding to β_1, β_2, β_3 and β_4 are proportional respectively to the numbers 100, 55, 16 and 9; the voltage applied to the tube in this experiment being 24,800 volts.

The ratio of intensities of the two α emission lines possesses special theoretical interest. Sommerfeld[5] has developed a theory which predicts the relative intensity of certain lines in the spectrum. By applying the quantum theory to electron orbits that do not lie in the same plane he finds that the number of postitions which the plane of an orbit can occupy is greater by unity than the quantum number associated with the electron's angular coördinate. According to a line of reasoning due to Bohr[11] one of these positions is in a certain sense dynamically impossible, which reduces the number of possible positions the plane of the orbit can occupy by unity. According to Sommerfeld's theory all the possible positions of the orbit's plane are equally probably, and hence the intensity of the line is proportional to the number of possible positions.

In the case of the α lines in the K series this theory agrees very well with the facts.[8] The α lines in the L series, however, do not have the relative intensity predicted by the theory. The elctrons producing these α lines fall from M orbits into the same L_1 orbit. Those producing the α_1 line fall from the first, M_1, orbit, which has three quanta associated with the angular coördinates, while those producing the α_2 line come from the orbit M_2, which has only two quanta associated with the angular coördinates. According to Sommerfeld, therefore, the relative intensities should be in the ratio of 4 to 3. As a matter of fact, the ratio is very much larger than this. The curve in figure 3 represents the two α lines in the L series of tungsten. The ratio of the heights of the two peaks is about 10 to 1. According to Bohr's "analogue principle" the transfer from M_2 to L_1 could not take place, unless the atom were in a field of force. This may account for the weakness of the α_2 line.

[1] Siegbahn and Friman, *Phil. Mag., London*, April, 1916 (403); and Nov. 1916 (497).
[2] de Broglie, *J. Physique, Paris*, May–June, 1916 (161).
[3] Duane and Patterson, *Proc. Nat. Acad Sci., Washington*, Sept. 1920.
[4] Webster and Clark, *Ibid.*, March, 1916; and Webster, *Ibid.*, Jan., 1920.
[5] Sommerfeld, *Atombau und Spektrallinien*.
[6] Duane and Patterson, *Physic. Rev., Ithaca*, 1920.
[7] Duane and Hu, *Ibid.*, Oct., 1919 (369).
[8] Duane and Stenström, *Ibid.*, April, 1920 (329); and *Proc. Nat. Acad. Sci. Washington*, Aug., 1920.
[9] Overn, *Physic. Rev.*, Aug., 1919 (137).
[10] Siegbahn, *Phil. Mag.*, Nov., 1919 (639).
[11] Bohr, *Copenhagen Academy*, 1918.

ON A DIFFERENTIAL EQUATION OCCURRING IN PAGE'S THEORY OF ELECTROMAGNETISM

By H. Bateman

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Communicated by R. A. Millikan, June 7, 1920

In a recent article Mr. Leigh Page[1] has generalized the electromagnetic equations by introducing the idea of a rotation of the field round a moving electric pole. A line of electric force is supposed to be the locus of points traveling with constant velocity, c, in directions which vary according to a law giving rise to three differential equations of type

$$k\frac{dl}{d\tau} = c(f + vr - wq) - m(c^2r - gu + fv) + n(c^2q - fw + hu) - cl[l(f + vr - wq) + m(g + wp - ur) + n(h + uq - vp)], \quad (1)$$

where (cl, cm, cn) are the component velocities of the point which leaves the electric pole at time τ; (u, v, w), (f, g, h) and (p, q, r) are the components of the velocity, acceleration and angular velocity of the pole at this instant. The quantity, k, is defined by the equation $k = c^2 - u^2 - v^2 - w^2$. These equations are of a type considered in my book on *Differential Equations* (Ex. 7, p. 71) and can be reduced to two Riccatian equations by means of the substitutions

$$s = \frac{l + im}{1 + n}, \sigma = \frac{l - im}{1 + n}, i = \sqrt{-1}. \quad (2)$$

The lines of force can consequently be found by solving a Riccatian equation just as in the usual electromagnetic theory.[2]

It should be noticed that Page's expressions for the electric and magnetic vectors may be written in the forms

$$\mathbf{E}_x = \frac{1}{c}\frac{\partial(\tau, \sigma)}{\partial(x, t)} - \frac{\partial(\tau, \theta)}{\partial(y, z)}, \qquad \mathbf{H}_x = \frac{\partial(\tau, \sigma)}{\partial(y, z)} + \frac{1}{c}\frac{\partial(\tau, \theta)}{\partial(x, t)}, \quad (3)$$

where

$$\theta = \frac{ec^3}{4\pi k}\frac{(x - \xi)p + (y - \eta)q + (z - \zeta)r}{c^2(t - \tau) - u(x - \xi) - v(y - \eta) - w(z - \zeta)},$$

$$\sigma = \frac{e}{4\pi}\frac{(x - \xi)f + (y - \eta)g + (z - \zeta)h + k}{c^2(t - \tau) - u(x - \xi) - v(y - \eta) - w(z - \zeta)}, \quad (4)$$

e is the electric charge associated with the moving electric pole, (ξ, η, ζ) are the coördinates of the pole at time τ and τ is defined in terms of (x, y, z, t) by the equation

$$[x - \xi(\tau)]^2 + [y - \eta(\tau)]^2 + [z - \zeta(\tau)]^2 = c^2(t - \tau)^2. \quad \tau < t.$$

The electromagnetic field is thus obtained by superposing on the usual electromagnetic field of a moving electric pole a field derived from a type of field, which I have considered elsewhere,[3] by writing **H** for **E** and $-\mathbf{E}$ for **H**. The properties of this additional field may be derived at once from the properties of the field described previously by simply interchanging the words magnetism and electricity.

If (f, g, h) are no longer restricted to be the components of the acceleration and k is no longer restricted to be $c^2-u^2-v^2-w^2$, the formulae (3), (4) still specify an electromagnetic field in which a line of force is the locus of points moving with velocity c in directions specified by a set of equations of type (1) and these equations can still be reduced to Riccatian equations by means of the substitution (2).

In a field of this more general type both electricity and magnetism travel away from the electric pole with the velocity of light while in the case of a field of Page's type there is an emission of magnetism but no emission of electricity. There is, however, no magnetic charge associate with the moving electric pole.

In a field of this general type it is possible when $f = g = h = p = q = r = 0$ for the points which trace out a line of force to move in one constant direction which is the same for all and for there to be no radiation of energy even through the electric pole has an acceleration.[3] It is possible, then, for an electric pole of this more general type to describe a circular orbit without radiating energy, as in Bohr's theory of the hydrogen atom. In the transition from one circular orbit to another the angular velocity (p, q, r) may be different from zero and there may be radiation of the type described by Page.

The weak point of the present theory is that it requires the presence of electric and magnetic currents in the space surrounding the electric pole. This space in fact contains a continuous distribution of electric and magnetic particles which have been emitted from the moving pole and are travelling along straight lines with the velocity of light.

[1] *Proc. Nat. Acad. Sci.*, **6**, March, 1920 (115).

[2] *Amer. J. Math.*, **37**, 1915 (192); see also Murnaghan, F. D., *Ibid.*, **39**, 1917; and *Johns Hopkins Circular*, July, 1915.

[3] *Proc. London Math. Soc.*, (Ser. 2) **18**, 1919 (95), paragraph 10.

A NEW PROOF OF A THEOREM DUE TO SCHOENFLIES

By J. R. Kline

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Communicated by E. H. Moore, July 7, 1920

In a paper recently published in the Transactions of the American Mathematical Society, Professor Robert L. Moore[1] proved the following theorem: If *ABCDA* is a closed curve, then there exist two sets of arcs α_1 and α_2 such that (1) each arc of α_1 lies wholly within *ABCDA* except

that its end-points are in AB and CD, (2) each arc of α_2 lies wholly within $ABCDA$ except that its end-points are on BC and DA, (3) each point of $ABCDA$, except A, B, C, and D, is an end-point of just one arc of α_1 or of just one arc of α_2, (4) through each point within $ABCDA$ there is just one arc of α_1 and just one arc of α_2, (5) each arc of α_1 has just one point in common with each arc of α_2. It is the purpose of the present note to show how this theorem may be used to establish the following theorem due to Schoenflies[2]—*Suppose the closed curves J_1 and J_2 are in continuous one-to-one correspondence under a transformation π. Let R_i $(i = 1, 2)$ denote the point set $J_i + I_i$, the interior of J_i. Then there exists a continuous one-to-one correspondence π' between the points of R_1 and R_2, such that points of J_1 and J_2 correspond as under the transformation π.*

Proof.—Let A_1, B_1, C_1 and D_1 be four points on the closed curve J_1 such that A_1 and C_1 separate B_1 and D_1 on J_1. Let $\pi(A_1) = A_2$, $\pi(B_1) = B_2$, $\pi(C_1) = C_2$ and $\pi(D_1) = D_2$. It may easily be proved, with the use of the definition of a simple closed curve and the properties of continuous one-to-one correspondences, that A_2 and C_2 separate B_2 and D_2 on J_2. Cover R_i $(i = 1, 2)$ with sets of arcs $(\alpha_{i,1})$ and $(\alpha_{i,2})$ having the properties described in Professor Moore's theorem. Consider the square whose vertices are A (0, 0), B (0, 1), C (1, 1) and D (1, 0). There exists a continuous one-to-one correspondence Σ between the points of that arc of J_1 from A_1 to B_1 which does not contain C_1 and the points of the interval AB of the Y axis, such that $\Sigma(A_1) = A$ and $\Sigma(B_1) = B$. Likewise there exists a continuous one-to-one correspondence Σ' between the points of that arc of J_1 from A_1 to D_1, which does not contain B_1, and the interval AD of the X-axis such that $\Sigma'(A_1) = A$ and $\Sigma'(D_1) = D$. Let us assign to every point P_1 of R_1 coördinates (x'_{P_1}, y'_{P_1}) in the following manner: Suppose P_1 is a point of R_1 lying neither on the interval A_1B_1 nor on the interval C_1D_1 of J_1. Let x_1 denote the end-point on A_1D_1 of that arc of $(\alpha_{1,2})$ which contains P_1. Then x'_{P_1} shall be the abscissa of $\Sigma'(X_1)$. If H_1 is a point of R_1 on the interval A_1B_1 of J_1, then x'_{H_1} is zero, while, if K_1 is a point of R_1 on the interval C_1D_1 of J_1, x'_{K_1} is one. Suppose Q_1 is a point of R_1 lying neither on the interval A_1D_1 nor on the interval B_1C_1 of J_1. Let Y_1 denote the end-point on A_1B_1 of that arc of $(\alpha_{1,1})$ which contains Q_1. Then y'_{Q_1} shall be the ordinate of $\Sigma(Y_1)$. If M_1 is a point of R_1 lying on the interval A_1D_1 of J_1, then y'_{M_1} is zero; while, if M_1 is on the interval B_1C_1 of J_1, then y'_{M_1} is one. We shall say that the point P_1 of R_1 corresponds to the point P of R, if and only if, $x'_{P_1} = x_P$ and $y'_{P_1} = y_P$. In this manner, a continuous one-to-one correspondence Γ is defined transforming the points of R into the points of R_1.

If H_2 is a point of the interval A_2B_2 of J_2, then we shall say that H_2 corresponds to the same point of AB as $\pi^{-1}(H_2)$ corresponds to under Σ^{-1}, while M_2, a point of A_2D_2, corresponds to the same point of AD as $\pi^{-1}(M_2)$ corresponds to under $(\Sigma')^{-1}$. In this manner, we define con-

tinuous one-to-one correspondences, T and T', between the points of A_2B_2 and AB and between the points of A_2D_2 and AD, respectively. We shall assign to every point P_2 of R_2 coördinates (x''_{P_2}, y''_{P_2}) in the following manner. Suppose P_2 is a point of R_2, lying neither on the interval A_2B_2 nor on the interval C_2D_2 of J_2. Let X_2 be the end-point on A_2D_2 of that arc of $(\alpha_{2,2})$ which contains P_2. Then x''_{P_2} shall be the abscissa of $(T')^{-1}X_2$. If H_2 is a point of R_2 on the interval A_2B_2 of J_2, then x''_{H_2} is zero, while, if H_2 is a point of R_2 on the interval C_2D_2 of J_2, then x''_{H_2} is one. Suppose Q_2 is a point of R_2, lying neither on the interval A_2D_2 nor on the interval B_2C_2 of J_2. Let Y_2 denote the end-point on A_2B_2 of that arc of $(\alpha_{2,1})$ which contains Q_2. Then y''_{Q_2} shall be the ordinate of $(T)^{-1}Y_2$. If M_2 is a point of R_2 on the interval A_2D_2 of J_2, then y''_{M_2} is zero; while, if M_2 is a point of R_2 on the interval B_2C_2 of I_2, then y''_{M_2} is one. We shall say that a point of P_2 of R_2 corresponds to a point P of R if and only if, $x''_{P_2} = x_P$ and $y''_{P_2} = y_P$. In this manner a continuous one-to-one correspondence Γ' is defined transforming the points of R into the points of R_2. It should be noted that the correspondence $\Gamma'\Gamma^{-1}$ is a continuous one-to-one correspondence transforming the points of R_1 into the point of R_2 in such a manner that the points of $B_1A_1D_1$ and $B_2A_2D_2$ correspond as under π. This is, however, not necessarily true of the points of $B_1C_1D_1$ and $B_2C_2D_2$.

Let P_2 be any point of A_2B_2, while $\pi^{-1}(P_2) = P_1$. Let Q_1 denote the end-point on C_1D_1 of that arc of (α_{11}) which contains P_1. Let $\pi(Q_1) = Q_2$. Let $(\Gamma')^{-1}(Q_2) = Q$. Let PQ denote the interval from P to Q of the straight line joining P and Q, where $P = (\Gamma')^{-1}(P_2)$. Let $\Gamma'(PQ)$ be the arc P_2Q_2. Do this for every point of A_2B_2. Then let P'_2 denote any point of A_2D_2 and let $\pi^{-1}(P'_2) = P'_1$. Let Q'_1 denote the end-point on B_1C_1 of that arc of $(\alpha_{1,2})$ which contains P'_1. Let $\pi(Q'_1) = Q'_2$, while $(\Gamma')^{-1}(P'_2) = P'$ and $(\Gamma')^{-1}(Q'_2) = Q'$. Let $P'Q'$ denote the interval between P' and Q' of the straight line joining these two points. Let $\Gamma'(P'Q')$ be the arc $P'_2Q'_2$. Do this for every point P'_2 of A_2D_2. In this manner two new sets of arcs (α'_{21}) and (α'_{22}) are obtained having all the properties of the original sets of arcs (α_{21}) and (α_{22}) and having the additional property (6) that if t is any arc of either set with end-points H_2 and K_2 and if $\pi^{-1}(H_2) = H_1$ and $\pi^{-1}(K_2) = K_1$, then one of the arcs of the set $(\alpha_{11}) + (\alpha_{12})$ joins H_1 and K_1.

Using these new sets of arcs, $(\alpha'_{2,1})$ and (α'_{22}), we may define in a manner similar to that used in defining Γ', a new continuous one-to-one correspondence Γ'' transforming the points of R into the points of R_2. It is clear that $\Gamma''(\Gamma)^{-1}$ is a continuous one-to-one correspondence transforming the points of R_1 into the points of R_2 in such a manner that the points of J_1 and J_2 correspond as fixed by π.

[1] Cf. Moore, R. L., *Trans. Amer. Math. Soc.*, **20**, 1919 (176).

[2] Cf. Schoenflies, A., *Math. Ann. Leipzig*, **62**, 1906 (324).

ARE THE SUPERIOR CERVICAL GANLIA INDISPENSABLE TO THE MAINTENANCE OF LIFE?

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The sympathetic nerve fibres which originate in the thoracic part of the spinal cord terminate in the superior cervical ganglia, from which separate bundles of nerve fibres go to the various parts of the head. Experiments with stimulation of the central end of the cervical sympathetic nerves revealed that these nerve fibres control various functions localized in the head. For instance, to mention the more important ones, the contraction and dilatation of the bloodvessels of various parts of the head, the constriction and dilatation of the iris and the secretion of various glands. Section of the sympathetic nerve showed that some of these innervations are in a state of tonus, for instance, the pupils become constricted and the bloodvessels of the ear (to be seen best in rabbits) become dilated. By means of local application of nicotine to the ganglia which paralyzes postganglionic but does not affect preganglionic nerve fibres, Langley has shown that some of the postganglionic nerve fibres of the superior cervical ganglia are simply a continuation of the cervical sympathetic, preganglionic nerve fibres, while other postganglionic nerve fibres have their origin in nerve cells within the superior cervical ganglia.

About sixteen years ago we[1] showed that epinephrin (adrenalin) which, if administered by subcutaneous injection, or by instillation in the conjunctival sac, in normal animals, shows no influence upon the size of the pupil, produces promptly a dilatation of the pupil and a considerable reduction of its responsiveness to light, if the subcutaneous injection or the instillation were made in animals in which the corresponding ganglion was removed. To obtain this result the subcutaneous injection or instillation must be made about twenty-four hours in rabbits and forty-eight hours in cats and dogs after the removal of the ganglion. Cutting of the cervical sympathetic nerves has no such effect. These facts were best illustrated in an experiment in which the superior cervical ganglion was removed on one side and the sympathetic nerve was cut on the other side. A simultaneous instillation of adrenalin in both conjunctival sacs caused a dilatation of the pupil on the side in which the ganglion was removed, but not on the side in which merely the sympathetic nerve was cut. For that reason we designated this pupilary phenomenon as "paradoxical." These experiments revealed that in the normal state, and for twenty-four hours in rabbits, or for forty-eight hours in cats and dogs, after the removal of the superior cervical ganglion, the normal condition of the pupil is maintained by influences which take their origin in nerve cells within the superior cervical ganglion and not in the spinal cord.

In studying at various times certain phases of this experiment, I was sometimes annoyed by failures which were caused by early deaths of the animals. A survey of these failures revealed the fact that they took place in animals in which both ganglia were removed. A survey of the several changes in the functions which stimulation of the sympathetic nerves or section of these nerves bring about, shows that these changes, interesting as they are, affect only peripheral conditions, and do not endanger life. But some of the post ganglionic fibres of the superior cervical ganglion enter the "brain" before they reach the periphery. May not some of these fibres also terminate in some vital point within the brain or the medulla? It is true, in our previous experimental studies, death of the animal was rather an exception. But these studies were essentially confined to the removal of a ganglion on one side only. From experiments upon the vagus nerves we know that cutting of one vagus does not affect the life of the otherwise normal animal, while, when both vagi are cut, with few exceptions, the animal dies within a day or two after the operation. May we not meet with a similar result by the removal of both ganglia? I therefore started a series of experiments in which both superior cervical ganglia were removed.

This series of experiments was made chiefly on rabbits. Although in these animals the superior cervical ganglion is small, it offers the advantage that it is not intimately connected with the vagus nerve, as it is the case in dogs and cats; the ganglion in rabbits may therefore be readily removed without causing injury to the neighboring vagus nerve. (It is not superfluous to state that the present series of experiments were carried out last winter, beginning November 3rd.) To this date more than eighty rabbits were used for the study of various phases of our problem. I shall state the results briefly.

For the chief issue of our problem we used twenty-eight animals in which both superior cervical ganglia were simultaneously removed under ether anesthesia. Of these, twenty-five died as follows: Six died in less than twenty-four hours, that is, the animal which was operated before noon or in the afternoon was found dead next morning at about nine o'clock. Nine animals died in less than two days, five in less than three days, one in four, one in five, one in six, one in eight and one in ten days. The autopsies revealed the presence of pulmonary lesions in all these animals. Three animals survived the removal of the ganglia and were killed after many weeks or months and no pulmonary lesions were found.

Both ganglia were also removed in two cats which died in less than two days and showed pulmonary lesions.

At an early stage of the investigation we met with one failure in a cat; it survived for many days the removal of both ganglia. Suspecting that the operation might not have been successful, it occurred to me, that this might be tested by the action of adrenalin upon the pupil. In the

experiments with adrenalin,[2] it was observed that when a ganglion was incompletely removed, instillation or subcutaneous injection of adrenalin did not cause the characteristic upon the pupil. I therefore applied this method to the surviving cat and found indeed that an instillation in a conjunctival sac as well as a subcutaneous injection of adrenalin neither caused a dilatation of the pupil nor a retraction of the nictitating membrane, the latter being one of the striking effects of adrenalin in ganglionectomized cats. The animal was then killed on the tenth day after the operation, and it was established that neither of the ganglia were successfully removed, and that the lungs showed no lesions. Also in one surviving rabbit it was found that adrenalin caused no dilatation of the pupil of the right eye. The autopsy showed normal lungs, proved that the ganglion on the right side was only partly removed. Henceforth an adrenalin test was made on the pupils of practically all animals in which the ganglia were removed. This was also done in the above mentioned three rabbits in which the removal of the ganglia did not lead to the death of the animals. In one of these animals the effect of adrenalin was indeed insignificant; in the other two rabbits, however, the instillation of adrenalin brought a definite dilatation of both pupils. In some rabbits the removal was purposely restricted to one half, or a little more than one half, of one ganglion or of both. As a rule the part to be removed was first crushed and then torn away. In these cases the adrenalin test was made a day or two later; the result was negative, and the animal survived even when both ganglia were mutilated.

However, in testing the success of the operation by the use of adrenalin, it has to be borne in mind that while a negative result prove that the corresponding ganglion was incompletely removed, a positive adrenalin test does not necessarily prove that the ganglia were so completely removed as to bring about the death of the animal. It may be that the number of nerve cells the removal of which is sufficient to bring about a positive adrenalin reaction upon the pupil, is smaller than the number of cells which have to be removed in order to bring about the death of an animal. In other words, the presence and integrity of a small number of nerve cells may be sufficient to maintain life, while the balance of the cells serve only as *factors of safety*—a condition which is especially met with in glands with internal secretion. It follows that in the above mentioned surviving rabbits in which the adrenalin test was positive, we had no assurance that the ganglia were so completely removed as to bring about the death of these animals. (It has to be added that at the autopsy of the three animals neither a minute macroscopical nor a microscopical search was made for possible remnants of the ganglia.)

However this may be, *this series of experiments showed that in about ninety per cent of the animals, the operative removal of both ganglia proved to be fatal to the animals—an experiment which, as far as I know, was never*

made before. The question now arose: Definite as the result is, is it important, that is, does it prove that the ganglia are really important to the maintenance of life, or was the fatal outcome perhaps due only to the operative procedure and not to the indispensability of some principle provided by the ganglia? Pulmonary lesions were found at the autopsy after the removal of both ganglia; pulmonary lesions are found after cutting both vagi. The ganglia are in the very proximity of the vagus nerves. May not the operative procedure injure both vagi sufficiently to bring about a "vagus pneumonia?"

The following considerations and experimental facts speak against such an interpretation. The possibility of injuring the vagus nerves was taken into consideration at the very beginning of this study. As it was stated before, it was just on account of this possibility that rabbits were given the preference. In these animals the superior cervical ganglion is separate from the vagus nerves, and it may be removed without touching the nerves or pulling at them. With this requirement in mind, I tried to carry out the experiment with care. It is not quite probable that I should have failed on both sides in nearly all my attempts, and that the injury which I might inadvertently have afflicted to both vagus nerves should have been of such an extent as to be equal in effect to a complete section of the nerves. Furthermore, in several experiments one of the ganglia could not be found and only one ganglion was removed. In these cases in which a long search for the ganglion was made the tissues and the corresponding vagus nerve had to be quite severely manipulated. The animals nevertheless survived this procedure. Finally in some experiments, the lower half of both ganglia were removed, as mentioned before. Here the lower half was first crushed and then removed, which required more manipulations than the tearing out of the entire ganglion. These animals survived the operation without manifesting any untoward symptoms.

More direct and decisive evidence was obtained by stimulations of the central end of the vagus nerves after removal of the corresponding ganglion, and after removal of both ganglia. The vagus nerves carry centripetal as well as centrifugal nerve fibres. The centripetal fibres affect and control the respiration and cause a change in the state of blood pressure. Further, the vagus trunk and the superior laryngeal nerve, a branch of the vagus, carry fibres the stimulation of which causes reflex deglutition. The vagus nerve seems to contain also sensory fibres. The stimulation was effected by Faradic currents of various strengths. The procedure was as follows: One vagus nerve was cut and the normal effect of stimulation established; then the corresponding ganglion was removed and the nerve again stimulated, and finally, the other ganglion was removed and the vagus stimulated. I shall state the results very briefly.

Respiration.—Stimulation of the vagus nerve after removal of the cor-

responding ganglion, or of both ganglia, gave results similar to those obtained before the removal, that is, active respiration, passive respiration (inhibition of the diaphragmatic movements), acceleration of the respiration, or a tetanic standstill of the diaphragm in an inspiratory state, according to the strength of the stimulus used and the individuality of the animal. *Bloodpressure.*—Stimulation of the vagus nerve gave a definite rise, and stimulation of the depressor nerve gave a fall. Rise and fall after removal of the ganglia were—at least— as good as before the removal. *Deglutition.*—When the animal was not too deeply narcotized, stimulation of the vagus trunk produced irregularly one deglutition or more, and stimulation of the central end of the superior laryngeal nerve produced regularly a series of swallows. *Sensation.*—Finally, it may be added that when the animal was nearly out of ether, stimulation with a strong current brought out signs of sensation, which gave the signal for an increase of the anesthesia. *These experiments show unmistakably that the afferent nerve fibres within the vagus were in no visible way affected by the act of removing the ganglia.*

The Centrifugal Fibres.—The efferent nerve fibres of the vagus, which concern us here, are those which innervate the muscles of the larynx, the pharyngeal muscles and the muscular fibres of the esophagus. The laryngeal innervation after removal of the ganglia has not yet been studied. However, clear evidence was obtained that the motor nerve fibres attending the pharynx and the esophagus were functioning in a normal way. *Under moderate anesthesia the inferior pharyngeal muscles were seen to contract with the rise of the larynx, which accompanies the initial stage of deglutition, and the rise was followed by a normal peristaltic wave in the esophagus.*

The condition of the esophagus at the autopsy is another evidence that the vagus was not affected by the procedure of the removing of both ganglia. In the autopsies made on the twenty-five rabbits which died after the removal of both ganglia, the esophagus was found empty and contracted in fourteen cases; in the other cases the condition of the esophagus was not noted. In other words, in all the cases in which the esophagus was examined, it was found that after removal of the ganglia this tube was empty and contracted. On the other hand, in a series of eighteen rabbits in which both vagi were cut and in which nearly all animals died in less than twenty-four hours, at the autopsy the esophagus was found distended and filled with food in its entire length in thirteen animals; in the remaining five the esophagus was either only partly filled or entirely empty but was not exactly contracted. I wish to point out here that the upper part of the esophagus is partly innervated by pharyngeal nerves which remain intact when both vagi are cut in the neck. Moreover, food may be driven out by the rigor of the esophagus which, after the rigor passes away, may be relaxed again. At any rate, *it is a noteworthy fact that after the removal of the ganglia the esophagus was found*

empty and contracted in all cases in which the state of that tube was looked for, while in thirteen out of eighteen animals in which both vagi were cut, the entire esophagus was distended and full of food, a fact which speaks for the contention that the vagus nerves have not been affected by the removal of the ganglia.

From the foregoing evidences, I am inclined to conclude *that the death of the animals in which both ganglia were removed was due to the removal of these organs—if I may call them so—and not due to the procedure of the operation. In other words, the superior cervical ganglia contain a principle which is essential for the maintenance of life.*

As to the nature of the pulmonary lesions which were found after death following the removal of both ganglia, I shall only say that nearly in all instances there was a bronchopneumonia which was mostly located in the upper and middle lobes, and a pulmonary edema located mostly in the lower lobes of the lungs. In a few cases there were large and small abscesses in some parts of the lungs, and in some lungs there were plugs in the small bronchi which were apparently connected with the abscesses and which consisted of fibrin, pus and foreign material. In a few instances there were also serous or purulent pleurisies. I shall not attempt to enter into further details. Furthermore, I shall not discuss the question whether there is any difference between the pulmonary lesions observed after the removal of both ganglia and the lesions found in the lungs after section of both vagi. This I can the more afford to do, since the centuries' old question as to the nature of the pulmonary lesions after section of both vagi was recently opened up again by the important researches of Schafer[3]), and the entire question will surely have to be a subject of further investigations.

As to the nature of the mechanism by which the ganglia may control life, I shall, for the present, offer only a hypothesis. At the beginning of this communication, I called attention to the fibres from the superior cervical ganglia which enter the "brain," and put forward the question, whether some of these fibres may not terminate in a vital part of the medulla oblongata. The hypothesis answers this question in the affirmative, namely, that the fibres reach the respiratory centre in which they exert a controlling influence upon the coördinate antagonistic activities of the laryngeal muscles in the function of respiration. As we know, the glottis becomes wider during inspiration and narrower during expiration and is otherwise changing its configuration in the different respiratory phases. The larynx is provided with extrinsic and intrinsic muscles which act alternately as adductors and abductors and are innervated by different fibers running in the trunks of the laryngeal nerves. According to the law which I termed *Antagonistic Innervation* or the law of *Reciprocal Innervation* (Sherrington), the adductor muscles of the larynx become inhibited during the contraction of the abductors, and the latter become inhibited during

the contraction of the adductors. Such complicated continuous muscular activities suggest the control of a centre. This central control is maintained in a state of tonus. The hypothesis assumes that the tonus of the central laryngeal control is maintained by impulses coming continuously from the superior cervical ganglia. Some time after the removal of these ganglia the respiratory laryngeal centre loses its tonus and its coördinating activity upon the laryngeal muscles during respiration. The loss of coördination gives rise to results similar in some way to those consequent upon cutting of the vagus nerves. The difference between the two conditions is, that in cutting the vagus nerves, the *peripheral (motor) innervation of all the muscles* are simultaneously abolished, while after removal of the ganglia *only the central control* is eliminated. In this case impulses may still reach the laryngeal muscles from the respiratory centre, but they will now contract in an incoördinate and harmful manner. Future experiments may reveal that the behavior and appearance of the glottis after removal of the ganglia differ from its behavior and appearance after cutting of the vagi.

This supposed reflex action of the ganglia upon the control of the antagonistic contraction of the laryngeal muscles is somewhat parallel to the action of the same ganglia upon the peripheral organs, for instance, upon the iris. In this instance it controls the width of the pupil by causing either a contraction of the dilator muscle simultaneously with a relaxation of the sphincter pupilae, or, it causes a constriction of the sphincter with a simultaneous relaxation of the dilator.

As to the nature of the origin of the impulses, I compared above the ganglia with the glands of internal secretion. I append here a very brief report of a few experiments, the making of which was stimulated by the mentioned comparison. In four rabbits both superior cervical ganglia were torn away from their upper connections, but were left connected with the sympathetic nerves. In addition, two ganglia from another rabbit were placed deep in the wound before closing it. *All four animals survived many weeks* and were killed later by chloroform; the lungs were found normal. A fifth rabbit was killed six days after the operation in an experiment in which by accident only one foreign ganglion was added. At the autopsy a small abscess was found in the right middle lobe. Probably, the animal would have recovered. These few experiments do not permit any definite conclusion; but the findings are suggestive. If further experimentation should give similar results, the question would be obvious: *whether other sympathetic ganglia also possess some sort of an internal secretion.*

[1] Meltzer, S. J., and Auer, Clara Meltzer, "Studies on the Paradoxical Pupil-Dilatation Caused by Adrenalin. I.—The Effect of Subcutaneous Injections and Instillations of Adrenalin upon the Pupils of Rabbits," *Amer. J. Physiol.*, **11**, 1904 (28–36); Meltzer, S. J., "Studies on the Paradoxical Pupil-Dilatation Caused by Adrenalin.

II.—On the Influence of Subcutaneous Injections of Adrenalin upon the Eyes of Cats after Removal of Superior Cervical Ganglion," *Ibid.*, **11**, 1904 (37–39); Meltzer, S. J., and Auer, Clara Meltzer, "Studies on the Paradoxical Pupil-Dilatation Caused by Adrenalin. III.—A Discussion of the Nature of the Paradoxical Pupil-Dilatation Caused by Adrenalin."

[2] Meltzer, S. J. and Auer, Clara Meltzer, l. c. (30).

[3] Schafer, E. Sharpey, *Quart. J. Exper. Physiol.*, **12**, 1919 (231–301).

OBSERVATIONS ON THE BODY TEMPERATURE OF DRY COWS

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While a knowledge of the normal fluctuations in the body temperature of cattle is of importance to the veterinarian insofar as it may help to detect diseased conditions, it assumes special significance when metabolism experiments on cattle are made with the respiration calorimeter. By means of the latter, the heat eliminated by the animal during a certain period of time, as well as the gaseous exchange between the animal and the atmosphere that surrounds it, is measured. But in order to determine accurately the heat production as distinguished from the heat elimination a correction must be applied for any storage or loss of heat by the animal body and failure to do so might introduce an appreciable error. For example, if at the end of an experimental period the body temperature of an animal weighing 400 kilograms differs by 1° C. from that at the beginning, it means a storage or loss of heat by the body amounting to 332 calories (assuming the specific heat of the body to be 0.83). The data on record regarding the body temperature of farm animals are unsatisfactory for the reason that the observations have not been repeated often enough under different conditions which are known to affect the temperature of man. In this investigation an attempt has been made to secure data regarding the extent and the course of the body temperature variations in cattle and to study some of the factors that might influence them. The investigation covers a period of some sixteen weeks. The animals were two dry cows used at the same time for metabolism experiments with the respiration calorimeter, in which strict control of feeding, environment, etc., is maintained. Several series of observations on the body temperature of the cows have been made with special reference to the study of the following:

(1) Variations in body temperature from about 7.30 a. m. to about 5.30 p. m., including the effect of water drunk.

(2) Thermal gradient in the body.

(3) Variations in body temperature from about 5 p. m. to about 7 p. m., including the effect of the feed.

(4) Effect of the act of defecation and of change in position.

(5) Daily variations in body temperature measured at exactly the same hour of the day.

The observations were made by means of carefully standardized clinical thermometers. During the greater part of the experiment one and the same thermometer was used, while for simultaneous readings of the rectal and vaginal temperatures a second one was used. To avoid any possible error that might be due to the use of two different thermometers, the latter were for a few days interchanged, i.e., the thermometer used one day for measuring the rectal temperature was used the following day for the vaginal and vice versa. This was further checked by using one and the same thermometer for both the rectal and vaginal temperature readings being taken one immediately after the other.

The observations on the thermal gradient were made at a depth of 4, 5, 6, and 7 inches. All other observations were made at a depth of 7 inches. The thermometers, which were originally five inches long, were accordingly lengthened by means of a piece of rubber tubing slipped a half inch or so over the upper end of the thermometer and over the string attached to the loop and stiffened by inserting inside the rubber tubing a strip of copper wire. This added portion was finally covered with adhesive plaster and a rubber ring tightly fitted at the end. This arrangement proved to be very satisfactory. The thermometer could be inserted to the desired depth and there was little danger of breaking it during the insertion. The thermometer was ordinarily allowed to stay in the rectum or vagina for about three minutes and the time and temperature recorded immediately upon its removal.

The results of the several series of observations may be summarized as follows:

(1) The rectal temperature was higher than the vaginal when measured at the same depth of 7 inches, showing an average excess of about 0.3° F. The relative values, however, varied under different conditions, but showed a trend toward parallelism.

(2) A fall in body temperature invariably followed the drinking of water. This fall varied directly with the quantity of water drunk. After the effect of the water drunk in the morning had been overcome, the temperature remained fairly constant till about 2.30 p. m. When no water was drunk the temperature was practically constant in the morning and in the afternoon till about 2.30 p. m.

(3) There was a gradual rise in temperature in the afternoon from about 2.30 p. m. to about 5.00 p. m.

(4) Eating of feed raised the body temperature slightly for about a half hour when the cows received a maintenance ration.

(5) The temperature of the rectum or vagina was decidedly higher when measured at a depth of 6 or 7 inches, than at a depth of 4 or 5 inches.

(6) There was no material difference in temperature between a depth of 6 inches and one of 7 inches, while there was a distinct difference of temperature between a depth of 4 inches and one of 6 inches, thus showing the unreliability of measurements of temperature at a depth of less than 6 inches.

(7) The standing as compared with the lying position of the animal had hardly any effect on the body temperature, but there was some indication that the temperature was slightly affected when measured shortly after the change in position had been made.

(8) There was no difference in body temperature when measured before or after defecation.

(9) Daily fluctuations in body temperature depend to a great extent on the individuality of the cow.

(10) A variation of 0.8° F. in the rectal temperature of the same animal was observed, when measured at the same hour of the day under identical conditions and outside the influence of water or feed, while under the influence of water a difference of 1.3° F. was observed at the same hour on two consecutive days.

A detailed report of the results will appear in the *Journal of Agricultural Research*.

ON THE STRUCTURE OF FINITE CONTINUOUS GROUPS WITH EXCEPTIONAL TRANSFORMATIONS

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Let $X_k \equiv \sum_1^{r+q} i\, f_{ik}(x) \frac{\partial}{\partial X_i}$ were $k = 1, 2, \ldots, r, r+1, \ldots r+q$, be the differential symbols of a finite continuous group G of order $r+q$, and let $E_k \equiv \sum_1^{r+q} j \sum_1^{r+q} i\, \alpha_j C_{jki} \frac{\partial}{\partial \alpha_i}$ be the symbols of the operators of the adjoint of G. We then have

$$(X_i, X_j) = \sum_1^{r+q} k\, c_{ijk} X_k \qquad (i, j = 1, 2, \ldots r+q)$$

and $$(E_i, E_j) = \sum_1^{r+q} k\, c_{ijk} E_k,$$

where the c's are the so-called structural constants. We assume that

the group G has q ($\leq r$) *exceptional* infinitesimal transformations (Lie calls them "*ausgezeichnete*"), which for simplicity are taken to be

$$X_{r+1}, X_{r+2}, \ldots X_{r+q}.$$

Then

$$(X_i, X_j) = 0 \quad (i = 1, 2, \ldots r+q; j = r+1, \ldots r+q).$$

The purpose of this paper is to find the conditions to be imposed on the group G which would make the structural constants $c_{ijr+1}, \ldots c_{ijr+q}$ all zero for $i, j = 1, 2, \ldots r+q$.

Since the adjoint of G has the same structure as G itself, it follows that

$$(E_i, E_j) = \sum_1^r k \; c_{ijk} E_k \; (i, j = 1, 2, \ldots r).$$

Therefore we may say that if G contains q exceptional infinitesimal transformations, there exists another group, say G', with r essential parameters having the same structure as the group G.

We shall denote the operators of the adjoint of G' by the symbols

$$M_k = \sum_1^r j \sum_1^r i \; \alpha_j c_{jki} \frac{\partial}{\partial \alpha_i} \quad (k = 1, 2, \ldots, r)$$

The condition we impose on the adjoint of G' is that it shall have one invariant spread. From that follows that the nullity of the matrix formed by the coefficients of the r differential operators is equal to one, i. e., at least one of the minors of the determinant of the coefficients of order $r-1$ is not zero. But every minor of that determinant is also a minor of the determinant formed by the coefficients of the differential operators of the adjoint of G, therefore in this larger determinant there will be at least one non-vanishing minor of order $r-1$. Furthermore, for $\alpha_1 \ldots \alpha_r \ldots \alpha_{r+q}$ assigned, the symbolic equations

$$\left(\sum_1^{r+q} i \; \alpha_i \overline{X}_v, \sum_1^{r+q} i \; \beta_i \overline{X}_i \right) = 0$$

have the following $q+1$ independent solutions:

$$\beta_1 = \alpha_1, \ldots \beta_r = \alpha_r, \beta_{r+1} = \ldots = \beta_{r+q} = 0 \quad (1)$$

$$\beta_1 = \ldots = \beta_r = 0, \beta_{r+1} = 1, \beta_{r+2} = \ldots = \beta_{r+q} = 0 \quad (2)$$

$$\beta_1 = \ldots = \beta_{r+1} = 0, \beta_{r+2} = 1, \beta_{r+3} = \ldots = \beta_{r+q} = 0 \quad (3)$$

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$$\beta_1 = \ldots = \beta_{r+q-1} = 0, \beta_{r+q} = 1 \quad (q+1)$$

That shows that there will be in the large determinant at least one non-vanishing minor of order $r-q-1$. Therefore, the nullity of the matrix formed by the coefficients of the operators of the adjoint of G is equal to $q+1$, which means that the adjoint of G leaves invariant $q+1$

independent functions in $r + q$ variables. It should be observed, however, that one of those functions will also be invariant to the adjoint of G' and will consist of the first r variables alone. The following theorem may therefore be stated:

If the adjoint of G' has one invariant, the adjoint of G has $q + 1$ independent invariants, one of which is the invariant of the adjoint of G'.

We assume first that q of those invariants, not involving the one common to both, G and G', are all flats of order $r + q-2$ in the $r + q-1$ space and that their common intersection is an $r-1$ flat. It is then proven that if that flat does not pass through any of the points of the space of the adjoint of G, corresponding to the exceptional transformations of G, we can form new operators, such linear functions of the old ones, of which r, not involving the exceptional operators, form an invariant subgroup of G of order r. This means that the structural constants, whose last subscripts are greater than r, will all become zero. If, however, those q invariants are not flats but algebraic spreads, not necessarily of the same order, none of which passes through the points of the space of the adjoint of G described above, then by considering the polars of the q invariant spreads we prove that the structural constants which have for their last subscripts numbers greater than r can also be made all zero. If, finally, each of the q invariant spreads passes through only one of those points, not necessarily the same for all spreads, then the structural constants described above can also be made all zero.

Detailed proofs and references are given in a paper accepted for publication by the *Annals of Mathematics*.